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Short Communication

Parameter identification for a single-degree-of-freedom system using autoregressive moving average model: Application to cutting system

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Abstract

This paper presents parameter identification for the single-degree-of-freedom (SDOF) system. The proposed method uses an autoregressive moving average (ARMA) model and a bisection method with the free acceleration response of the system to an impact test. The method has been tested on the numerically integrated accelerations from a known SDOF system. The experiment has been conducted on cutting system with an end mill. The validity and advantages of the method are presented. The results show that the proposed method gives parameter identification with good quality for short accelerations using the ARMA model and the bisection method for the SDOF system.

1. Introduction

A good modeling method of the real system is important in engineering practice. The modeling method is to determine the parameters of the equation of motion for the system based on information contained in the system's dynamical response. There are several methods to identify the parameters of the system from the dynamical behavior. Tobias [1] modeled a single-degree-of-freedom (SDOF) system with the displacement response at each exciting frequency using an exciter. Badrakhan [2] considered the free vibrations of a SDOF system with combined viscous damping and Coulomb dry friction using only the amplitude decay of the displacement response of the system. Ying and Joseph [3] presented the identification of a stable linear system using polynomial kernels. Sekavcnik [4] discussed the identification of model damping parameters. Nonlinear systems are also approached in several different ways [5–9].

This paper proposes a modeling method of the mass, the damping coefficient and the stiffness of the linear SDOF system. It uses an autoregressive moving average (ARMA) model and a bisection method with the free acceleration response of the system to an impact test. The ARMA model obtains the system's equivalent viscous damping ratio and natural frequency from the measured accelerations, and the bisection method

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determines the system's mass, damping coefficient and stiffness. The method, as it will be seen, provides a direct and accurate means of obtaining the parameters of the equation of motion for the linear SDOF system.

The SDOF has being widely used in engineering practice. The proposed modeling method was experimentally applied to the cutting system because one of the fields that use the SDOF is cutting. The stability of machine-tool chatter forms a key factor in the determination of the rate of metal removal and is the cause of poor surface finish and annoying noise. In order to solve these kinds of problems, the cutting system is considered to be the linear SDOF system, and this system is used to analyze the chatter vibration [10–15] and to predict the dynamic surface topography [16–18]. So, the proposed method is applied to an end mill to see the validity and advantages.

In Section 2, the mathematical fundamentals of the proposed method are presented using the ARMA model and the bisection method. Section 3 presents parameter identification procedure. Section 4 presents the results from a simulation study with the numerically integrated accelerations from a known SDOF system. The application of the method to an end mill is presented in Section 5. Conclusions are given in Section 6.

2. Theoretical considerations

Scalar ARMA models can be fitted to any single series of data. When a n-DOF system is subjected to an impact force, the displacements, velocities, and accelerations can be modeled by ARMA models, and from these models the natural frequencies and damping ratios of the system can be obtained. The signals of the n-DOF system are modeled by an ARMA($2n$,$2n$–1) model, the details of this model being available in the literature [19]. A stationary stochastic signal, $X(t)$, can be represented by a continuous ARMA(n,m) model:

$$\begin{aligned}(1-\phi_1 D-\phi_2 D^2-\cdots-\phi_{n-1}D^{n-1}-\phi_n D^n)X(t)\\ =(1-\theta_1 D-\theta_2 D^2-\cdots\cdots-\theta_{m-1}D^{m-1}-\theta_m D^m)Z(t),\end{aligned}$$

$$m\leqslant n-1,\quad E[Z(t)]=0,\quad E[Z(t)Z(t-u)]=\delta(u)\sigma_a^2 \tag{1}$$

where $D=\mathrm{d}/\mathrm{d}t$ is the differential operator, $Z(t)$ is the white noise, E denotes the expectation operator, $\delta(u)$ is the Dirac delta function, $\phi_1,\phi_2,\ldots,\phi_n$ are the autoregressive parameters and $\theta_1,\theta_2,\ldots,\theta_m$ are the moving average parameters.

The model can be written as

$$X(t)=\frac{(1-\theta_1 D-\theta_2 D^2-\cdots-\theta_n D^m)}{(1-\phi_1 D-\phi_2 D^2-\cdots-\phi_n D^n)}=\frac{\Theta(D)}{\Phi(D)}Z(t). \tag{2}$$

The discrete characteristic equation of the model is

$$\lambda^n-\phi_1\lambda^{n-1}-\phi_2\lambda^{n-2}-\cdots-\phi_n=0. \tag{3}$$

The natural frequencies and the damping ratios are obtained from the discrete charateristic roots as

$$\omega_{ni}=\frac{1}{\varDelta}\sqrt{\frac{\{\ln(\lambda_i\lambda_i^*)\}^2}{4}+\left\{\cos^{-1}\left(\frac{\lambda_i+\lambda_i^*}{2\sqrt{\lambda_i\lambda_k^*}}\right)\right\}^2}, \tag{4}$$

$$\zeta_i=\sqrt{\frac{\{\ln(\lambda_i\lambda_i^*)\}^2}{\{\ln(\lambda_i\lambda_i^*)\}^2+4\left\{\cos^{-1}\left(\frac{\lambda_i+\lambda_i^*}{2\sqrt{\lambda_i\lambda_k^*}}\right)\right\}^2}}, \tag{5}$$

where $\varDelta$ is the sampling interval, and λ_i,λ_i^* are complex conjugates.

3. Parameter identification procedure

In order to obtain the mass m, the damping coefficient c and the stiffness k of the SDOF system, let us consider the differential equation of motion

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = \delta(t), \tag{6}$$

where $\delta(t)$ is the impact force. If the system is excited by an impact force as shown in Eq. (6), the accelerations of the system decay with time oscillating with a natural frequency and a damping ratio. When the accelerations of the system are acquired from an impact test, the natural frequency and the damping ratio of the system can be obtained from Eqs. (4) and (5), respectively.

The ARMA(2,1) model can obtain two of the characteristic values of the system: the natural frequency and the damping ratio. However, three parameters need to be obtained: m, c, and k. We can obtain these three parameters using the impact force and the bisection method. Using the natural frequency and the damping ratio obtained from the ARMA(2,1) model after the impact test, a new equation of motion can be written:

$$\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2 x(t) = \delta(t). \tag{7}$$

If the same impact force $\delta(t)$ as used in Eq. (6) excites Eq. (7), the natural frequency and the damping ratio of Eq. (7) are the same as those of Eq. (6), but the amplitudes of the accelerations are different. Multiplying by a mass m_0, that is assumed to be smaller than the real value, gives a new equation of motion:

$$m_0\ddot{x}(t) + 2\zeta\omega_n m_0\dot{x}(t) + \omega_n^2 m_0 x(t) = \delta(t). \tag{8}$$

The accelerations in Eq. (8) can be integrated numerically using the same impact force $\delta(t)$ and the same sampling frequency as those from Eq. (6). While gradually increasing the mass m_0, let us compare the root mean square (RMS) of the simulated accelerations in Eq. (8) with the RMS of the accelerations measured using Eq. (6). When the RMS of the simulated accelerations is within a tolerance value of the RMS of the measured accelerations, it can be considered that Eq. (8) is nearly the same as Eq. (6). The bisection method locates the value of m_0 quickly and easily, as the RMS of the acceleration in Eq. (8) decreases linearly with increasing m_0.

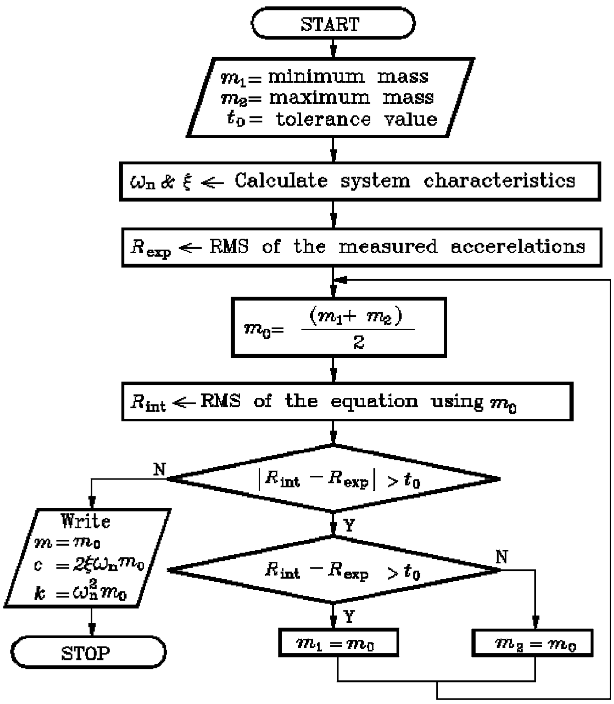

Fig. 1. The modeling procedure using the bisection method.

The modeling procedure can be summarized as follows (also see Fig. 1):

Step 1: Define a mass m_1 that is smaller than the real value, and a mass m_2 that is larger than it, and a tolerance value t_0 to use in the bisection method.
Step 2: Obtain the natural frequency and the damping ratio using the ARMA(2,1) model from the accelerations measured in the impact test.
Step 3: Obtain the RMS of the measured accelerations, R_{exp}.
Step 4: Set the mass $m_0 = (m_1 + m_2)/2$ and construct an equation of motion using Eq. (8), then integrate the accelerations of this equation using the same impact force and sampling interval as the experiment.
Step 5: Obtain the RMS of the integrated accelerations, R_{int}.
Step 6: If $|R_{\mathrm{int}} - R_{\mathrm{exp}}| > t_0$, go to Step 7, otherwise, finish the modeling procedure setting $m = m_0$, $c = 2\zeta\omega_n m_0$, $k = \omega_n^2 m_0$.
Step 7: If $R_{\mathrm{int}} > R_{\mathrm{exp}}$, set $m_1 = m_0$ and go to Step 4, otherwise, set $m_2 = m_0$ and go to Step 4.

4. Simulation study

To investigate the performance of the proposed method, a simulation study of a known vibratory system was carried out. The SDOF system is as follows:

$$5\ddot{x}(t) + 20\dot{x}(t) + 4000x(t) = \delta(t). \tag{9}$$

The accelerations were generated using the Wilson-θ method with an impact force of 1 N in Eq. (9). The sampling time was 0.005 s, a value shorter than the Nyquist sampling time of 0.0176 s, improving the accuracy of the parameters in the ARMA model, and therefore improving the accuracy of the calculated natural frequency and damping ratio. Fig. 2 shows the generated accelerations and Fig. 3 shows the FFT of the accelerations.

After the data were generated, they were treated as if the original system was unknown, and the system parameters were identified and compared with Eq. (9) using the method outlined in the previous section. The natural frequency and damping ratio were subsequently calculated using the estimated model parameters from

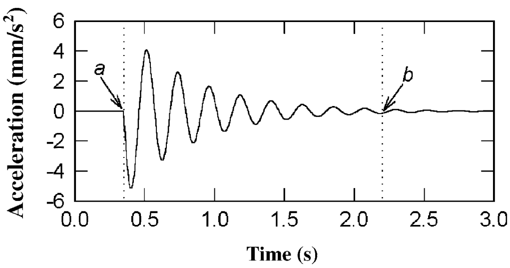

Fig. 2. The accelerations of a vibratory system.

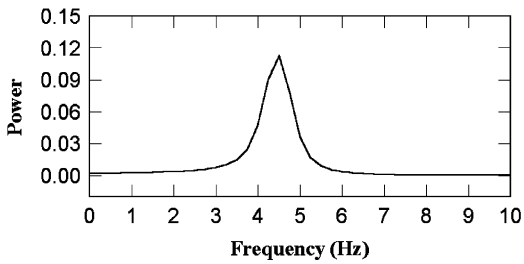

Fig. 3. The FFT of the accelerations.

Table 1
The ARMA(2,1) parameters and comparison of identified and theoretical system characteristics

The parameters of ARMA	ϕ_1 1.9605	ϕ_2 0.9803	θ_1 0.0044
Natural frequency	Identified 4.5078 Hz	Theoretical 4.5016 Hz	
Damping ratio	0.0702	0.0707	

Eqs. (4) and (5). The data between '*a*' and '*b*' in Fig. 2 were used for the ARMA(2,1), because the data outside the perimeter '*a*' and '*b*' do not relate to the system characteristics. The amplitude of the acceleration at '*b*' was 5% of the maximum amplitude, and all modeling of the ARMA in this research was performed in the same manner. Since the original system was in fact known, the natural frequency and damping ratio could be calculated directly from Eq. (9), and are also given in Table 1 for comparison.

The extracted values show a remarkable match with those calculated theoretically. Using the natural frequency and the damping ratio calculated using the ARMA(2,1) model, the equation of motion from Eq. (7) can be derived:

$$\ddot{x}(t) + 3.9766\dot{x}(t) + 802.2177x(t) = \delta(t). \tag{10}$$

For a complete modeling, the bisection method was utilized as shown in Fig. 1 with the following initial conditions: $m_1 = 0.01$, $m_2 = 1000$ and $t_0 = 0.0089$. The RMS of the accelerations equaled 0.89 and the tolerance value was set to 0.0089, 1% of the RMS value. Using these initial conditions, the program for finding these parameters converged to $m_0 = 5.0059$ after 12 iterations. The result of the modeling is as follows:

$$5.0059\ddot{x}(t) + 19.9064\dot{x}(t) + 4015.7737x(t) = \delta(t). \tag{11}$$

The modeled Eq. (11) is in good agreement with the known system of Eq. (9).

5. Experimental results: modeling of an end milling

The feasibility of the modeling procedure is illustrated by applying it to end milling. Fig. 4 shows the experimental setup used for the impact test. The spindle of a vertical milling machine can be assumed to be a rigid body, and the experiment was carried out on an end mill as shown in Fig. 4.

A vice fixed the end mill, the accelerometers being attached on the end mill. The impact hammer excited the end of the end mill, and the respective detectors measured the impact forces and the accelerations. The diameter of the end mill was 12 mm and fixed to a length of 80 mm. Fig. 5 shows the accelerations of the end mill and Fig. 6 is the FFT of the accelerations.

The measured impact force of the end mill was 157 N and the sampling frequency was 6000 Hz. The natural frequency and the damping ratio were calculated as in the previous section and are presented in Table 2 with the ARMA parameters.

The equation of motion can be written with the natural frequency and the damping ratio as in Eq. (7):

$$\ddot{x}(t) + 195.2\dot{x}(t) + 5.6370064 \times 10^7 x(t) = \delta(t). \tag{12}$$

The bisection method was used as in the previous section with the following initial conditions: $m_1 = 0.1$, $m_2 = 100$, and $t_0 = 0.001$. With these initial conditions, the program for finding parameters converged to $m_0 = 0.9$ after 17 iterations. The result of the modeling is as follows:

$$0.9\ddot{x}(t) + 175.8\dot{x}(t) + 5.07056987 \times 10^7 x(t) = \delta(t), \tag{13}$$

which means that $m = 0.9$ kg, $c = 175.8$ N/mm/s and $k = 5.07056987 \times 10^7$ N/mm.

To investigate the accuracy of the modeling, the accelerations were generated using the Wilson-θ method on Eq. (13), using an impact force of 157 N and a sampling frequency of 6000 Hz, the same values as the experiment. Fig. 7 shows the generated accelerations and Fig. 8 shows the FFT of the accelerations. Fig. 9

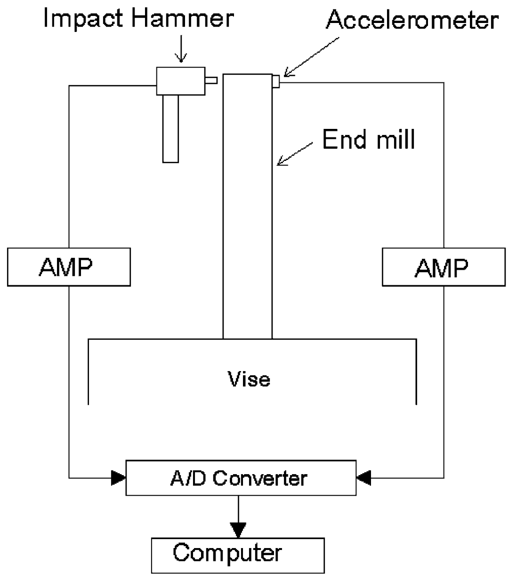

Fig. 4. Experimental setup.

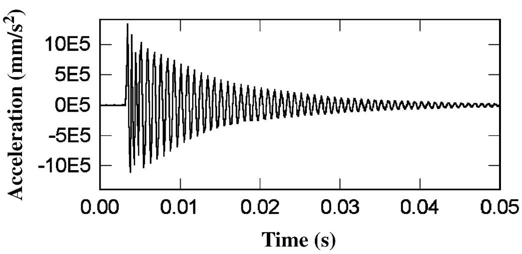

Fig. 5. The measured accelerations of the end mill.

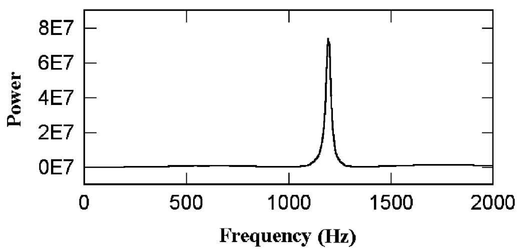

Fig. 6. The FFT of the measured accelerations.

Table 2
The ARMA(2,1) parameters and the characteristics of the end mill

The parameters of ARMA	ϕ_1	ϕ_2	θ_1
	0.6175	−0.9670	−0.0052
Natural frequency	1195 Hz		
Damping ratio	0.013		

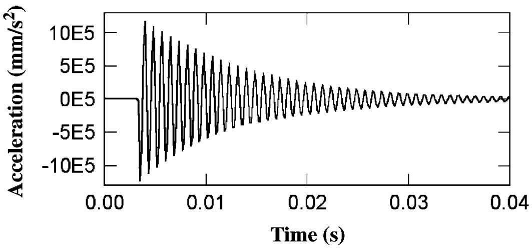

Fig. 7. The accelerations of the modeled system.

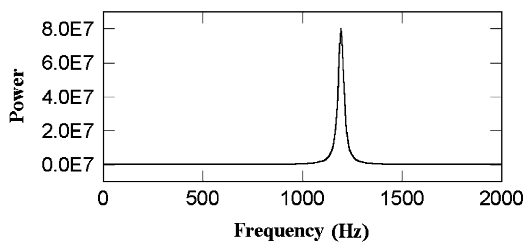

Fig. 8. The FFT of the generated accelerations.

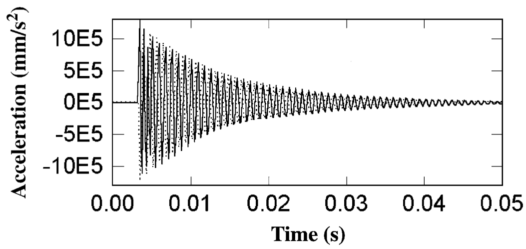

Fig. 9. The comparison of the accelerations in the end mill: experimental data (—), generated data (……).

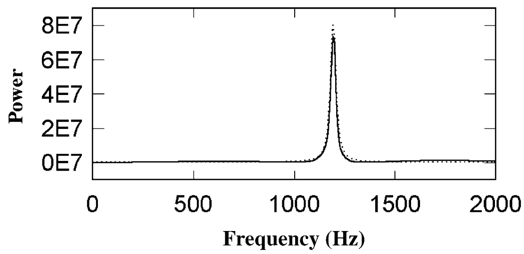

Fig. 10. The comparison of the FFTs in the end mill: experimental data (—), generated data (……).

shows a comparison of the accelerations from the experiment of Fig. 5 with the modeling of Fig. 7. Fig. 10 shows the FFTs of the accelerations. The results from the modeling are in good agreement with the experiment.

6. Conclusions

This paper presents parameter identification for the linear SDOF mechanical system using the ARMA model, and the bisection method with measured accelerations by the impact test is presented. The proposed method follows the concept of computing the mass, the damping coefficient and the stiffness of the linear SDOF system if the parameters are considered to be unknowns.

The natural frequency and the damping ration were obtained from the measured accelerations using the ARMA(2,1) model, and the mass, the damping coefficient and the stiffness of the SDOF system were determined using the bisection method with the measured impact force.

The simulation study was conducted on the numerically integrated accelerations from the known SDOF system. The proposed method experimentally applied to the cutting system with the end mill to demonstrate the validity and advantages. The simulation and experiment results show the proposed method offers parameter identification with good quality for the free acceleration response data of the SDOF system.

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